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"NEGENTROPY AND THE CONCEPTS OF FREEDOM, DEMOCRACY AND JUSTICE."

by

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Socio-Engineering Problems Reports are a series of manuscripts on the social relations of engineering and related philosophical questions dealing with the interaction of science and society. Distribution is limited to reviewers and discussion groups for criticism prior to consideration for possible publication.

This issue is a part of Chapter 10 of a proposed book, "Communication Theory in the Cause of Man." A short outline of the book plan is included as Appendix I.

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"Negentropy and the Concepts of Freedom, Democracy and Justice."

I. INTRODUCTION AND DEFINITIONS

A. OBJECTIVES.

This study is an examination of the usefulness of a concept -- negentropy -- from the physical sciences in bridging the gap between the two cultures of "science" and the "humanities." This material is offered as a "theoretical hypothesis" on the usefulness of the concept of entropy from the physical sciences as a tool of [illegible] value to social science.

The possibility that entropy in thermodynamics might belong both to the family of measurable quantities of science and the family of ideas such as beauty and melody was suggested many years ago by Eddington.(1) The development of the mathematical theory of communication by Shannon (2) in 1948 and the partially overlapping concepts of Cybernetics developed by Wiener (3,4) made it possible to move from a rough analogy to the consideration of more specific applications of quantities such as "information" or "negative entropy." In this paper I shall confine my discussion to the relevancy of the concept of negentropy to some important concepts in our civilization, namely "freedom," "democracy," and "justice."

I shall try to avoid the [illegible] that Raimon Lull ran into in the transition from [illegible] 14th centuries when he developed a rudimentary [illegible] was not content to lead mankind on the path of [illegible] machine which would be a very useful tool, but he [illegible] run too far ahead and tried to use his logic mac[illegible] theological arguments. His attempted application of [illegible] to questions of theological faith led to his being stoned [illegible] in Bugia, Morocco, in 1315.(5)

I wish to dedicate this paper to the memory of Simone Weil, a French mathematics teacher and scholar of the human [illegible] who [illegible] to acquire the necessary experience to bridge the gap between the sciences and the humanities in a way to help reconstruct human society. Her untimely death during World War II while on the staff of General DeGaulle's Free French prevented her from achieving her aims, but her notebooks and papers have been [illegible] to man, and she is [illegible] concerned with the problems of our [illegible] (6, 6A)

2. PLAUSIBILITY ARGUMENT FOR CHOICE OF ANALOGY

Biological systems [illegible] in some order [illegible] increasing [illegible]. The life process represents a local reversal of the degradation processes predicted by the second law of thermodynamics. The units of information are related to both the life process and to negative entropy (or negentropy)* in thermodynamics. Physically entropy can be defined as:

$S = k \ln W$ where k is the Boltzman constant of 1.38×10^{-16} erg/°K, "ln" means logarithm of, and W is the number of elementary states the system can be in.

In information theory the negentropy of a message of n symbols is:

$$H = -(P_1 \ln P_1 + P_2 \ln P_2 + \ldots + P_k \ln P_k + \ldots + P_n \ln P_n)$$

where P_k is the probability of occurrence of symbol k.

If we take the formula for information or negentropy and substitute the philosophical systems (or political systems) in place of the symbols of a message, then the probability of [illegible] of the respective philosophies among the population [illegible] is analogous to the probability of occurrence of the symbols in a message.

* The term "negentropy" was introduced by Leon Brillouin [illegible]

[illegible]

[illegible] though [illegible] adhere to the official philosophy [illegible] and this philosophy is number [illegible]

[illegible] = 0.

Thus the requirement that people adhere to an official philosophy is equivalent to a zero contribution to the negative entropy of the political system or the "life process" or the evolution toward a higher order of life. If we go back to our equation to see under what conditions leads to a maximum contribution to the negentropy of [illegible] when $p_1 = p_2 = \ldots = p_n$ [illegible] This corresponds to equal probability for each different philosophy: a condition approximating a democracy. After reviewing the derivations of the concepts, we shall make a more detailed [illegible]

c. DEFINITIONS.

Before proceeding with this study it is important to review the dictionary definitions of the principal words we are using. The definitions in Table I are from Webster's Seventh New Collegiate Dictionary.(7) Where a definition uses directly another word, the definition of the second word is also included in Table I.

My plan for this paper is to first review some elementary properties of finite sets of discrete messages that might be sent over a telegraph line. These simple examples will illustrate the the relationship of the probabilities of different messages being sent, the negentropy component of the individual messages, and the negentropy of the set of messages. Then I shall consider a hypothetical world divided into six countries of 100,000 people in each country. These six countries will have a range of social orders from ideal democracy to a dictatorship. I shall assume a set of probability distributions for the chances of an individual having a measure of freedom in these different social orders.

Then I shall make the hypothesis that the negentropy of the set of probabilities of freedom in a country or sub-system is a measure of the "democracy" of the sub-system. The next step is to compare these numerical results with our common sense rating of social systems in order of increasing amount of "democracy." If there is consistency we can assume the relationship between "negentropy" and "democracy" is a useful hypothesis, even though

TABLE I: DEFINITIONS

Definitions from Webster's Seventh Collegiate Dictionary (1963)

ENTROPY.

- 1a: a measure of the unavailable energy in a closed thermodynamic system so related to the state of the system that a change in the measure varies with change in the ratio of the increment of heat taken in to the absolute temperature at which it is absorbed
- 1b: a measure of the disorder of a closed thermodynamic system in terms of a constant multiple of the natural logarithm of the probability of the occurence of a particular molecular arrangement of the the system that by suitable choice of a constant reduces to the measure of unavailable energy
- 2: a measure of the amount of information in a message that is based on the logarithm of the number of possible equivalent messages
- 3: the degradation of the matter and energy in the universe to an ultimate state of inert uniformity

FREEDOM

- 1: the quality or state of being free: as
  - a: the absence of necessity, coercion, or constraint in choice or action
  - b: liberation from slavery or restraint or from the power of another: INDEPENDENCE
  - c: EXEMPTION, RELEASE
  - d: EASE, FACILITY
  - e: FRANKNESS, OUTSPOKENNESS
  - f: improper familiarity
  - g: boldness of conception or execution
  - h: unrestricted use
- 2a: a political right
- 2b: FRANCHISE, PRIVILEGE

FREE

- 1a: having the legal and political rights of a citizen
- 1b: enjoying civil and political liberty

(See also definitions 2 through 15)

p. 2B

TABLE I (continued): DEFINITIONS

DEMOCRACY

1a: a government by the people, esp.: rule of the majority

1b: a government in which the supreme power is vested in the people and exercised by them directly or indirectly through a system of representation usu. involving periodically held free elections

2: a political unit that has a democratic government

3 cap: the principles and policies of the Democratic party in the U.S.

4: the common people esp. when constituting the source of political authority

5: the absence of hereditary or arbitrary class distinctions or privileges.

JUSTICE

1a: the maintenance or administration of what is just esp. by the impartial adjustment of conflicting claims or the assignment of merited rewards or punishments

1b: JUDGE

1c: the administration of law; esp.: the establishment or determination of rights according to the rules of law or equity

2a: the quality of being just, impartial, or fair

2b(1): the principle or ideal of just dealing or right action

2b(2): conformity to this principle or ideal: RIGHTEOUSNESS

2c: the quality of conforming to law

3: conformity to truth, fact, or reason: CORRECTNESS

JUST

1a: having a basis in or conforming to fact or reason: REASONABLE

1b archaic: faithful to an original

1c: conforming to a standard of correctness: PROPER

2a(1): morally right or good: RIGHTEOUS

2a(2): MERITED, DESERVED

2b: legally right

IMPARTIAL: not partial

PARTIAL : 1: inclined to favor one party more than the other: BIASED [illegible] contradictions between the hypothesis and accepted facts. For further discussion of hypotheses in science, refer to either W. K. H. Panofsky (8) or Gerald Holton (9).

The next question is whether the concept of negentropy can also be useful in measuring some longer range variable which might be an extension of the concept of "justice." The static measure of deomcracy at a given time may be overshadowed by other factors such as the degree of organization or stability, if an ideal democracy cannot survive an attack from a dictatorship. It is felt that additional parameters not accessible in this study will be required in order to develop an extended concept of justice* which would include the ability of a more democratic country to defend itself. However we shall include some preliminary remarks on how this problem might attacked, but the study has not reached the stage where it is possible to put numbers on the relative "justice" of the total system including the interaction with time between the six hypothetical countries considered in this paper.

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* At present I shall call this extended concept of justice "dynamic-justice," implying the definitions of "justice" in Table I (items Ia and Ic) represent "static-justice." A possible qualitative test of "dynamic-justice" is discussed in another manuscript in preparation, entitled "Four Philosophical 'Tools' For Improving Our Insights Regarding the Problems of Disarmament," SEPR no. 19.

2. Negentropy.

In this section we shall review briefly the concepts of the entropy of probability distributions. These equations apply to Webster's definition 2 in Table I. The examples in this section apply to sets of messages which might be sent over a telegraph line. For those who want a brief introduction to Information Theory, I recommend the following books:

Colin Cherry, On Human Communication (10)
Ch. Two: Evolution of Communication Science-- An Historical Review
Ch. Five: On the Statistical Theory of Communication

R. Duncan Luce, Robert R. Bush, and J. C. R. Licklider, Deveopments in Mathematical Psychology (11)
Part I: The Theory of Selective Information and Some of Its Behavioral Applications

J. R. Pierce, Symbols, Signals and Noise (12)
Ch. I. The World and Theories
Ch. II. The Origins of Information Theory
Ch. III. A Mathematical Model
Ch. IV. Endoding and Binary Digits
Ch. V Entropy

The entropy of the set of messages is defined as:

$$I = \sum_{i=1}^{n} p_i \log_2 p_i \qquad (1)$$

where p_i is the probability that the i-th message will be sent. Since the probability p_i is a positive number between zero and one,

$$\log p_i \leq 0,$$

we can define the negentropy as minus the entropy,

$$H = -I, \qquad (2)$$

or

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i \qquad (3).$$

The choice of the base of the logarithm to the base two is arbitrary. For this study eq. (3) becomes:

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i = \sum_{i=1}^{n} p_i U_i \qquad (4)$$

where $U_i = -\log_2 p_i$ is sometimes called the "uncertainty." (*)

* David Middleton, Statistical Communication Theory. N.Y.: McGraw-Hill (1960), pp. 293-5.

Sample values of p_i, $-U_i$, and $U_i p_i$ are tabulated in Table II for a useful range of values. For convenience of the user, these parameters are plotted in Fig. I. There is a scale change in the center of Fig. I where the direction of the log-log paper reverses. The parameters p(x), U(x), and p(x)U(x) are plotted against p(x) on the left half and agianst [I-p(x)] on the right half. This choice of scale makes the curves asymptotic to straight lines for simpler graphical construction and application.

Curve (A) is the simple probability, p(x).

Curve (B) is the uncertainty, $U(x) = -\log_2 p(x)$.

Curve (C) is the product of curves (A) and (B), or the negentropy component H(x) corresponding to p(x).

If we have a set of two messages which can be sent over a telegraph line and their probabilities of being sent are p_1 and p_2, the total probability is $p_1 + p_2 = 1.0$. For example, if $p_1 = 0.1$ and $p_2 = 0.9$, we have from eq. (4):

$$H = -0.1 \log_2 0.1 - 0.9 \log_2 0.9$$

The uncertainty terms can be calculated or read off of Fig. I,

$$H = 0.332 + 0.137 = 0.469$$ Negentropy of the set of messages.

A curve for the total negentropy is plotted in Fig. 2A for all combinations of p_1 and p_2. The negentropy of the system is maximum for $p_1 = p_2 = 0.5$

For a set of three messages we have the condition

$$p_1 + p_2 + p_3 = 1.0$$

which means there are two independent variables, so we can use a two-dimensional plot to obtain equi-negentropy lines for the case of three messages. Note that the edges of the triangular coordinate plot in Fig 2B are the top projection of Fig. 2A. Equi-negnetropy lines for H= 0, 0.5, 1.0, 1.5, 1.585 are plotted in Fig 2B.

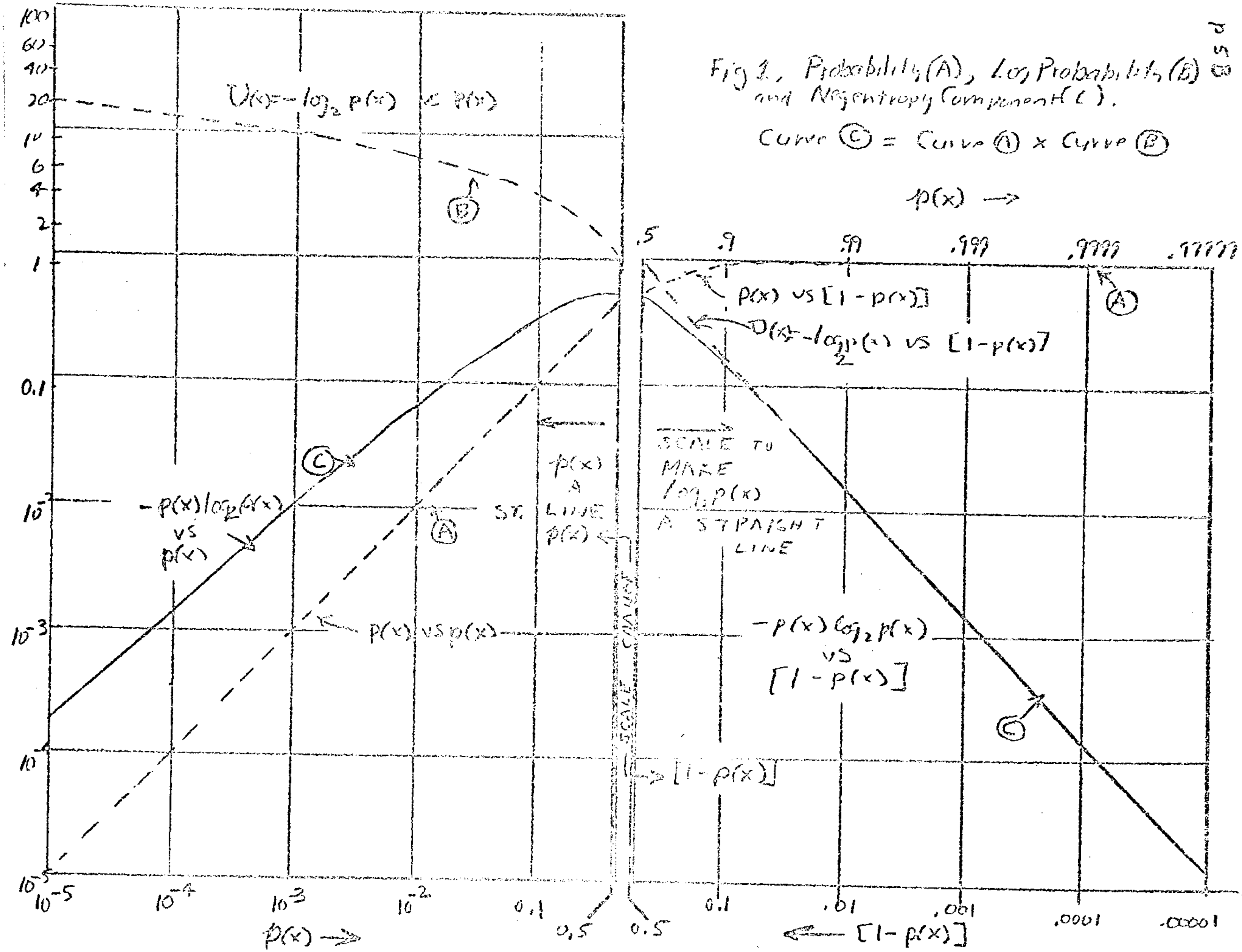
/5A

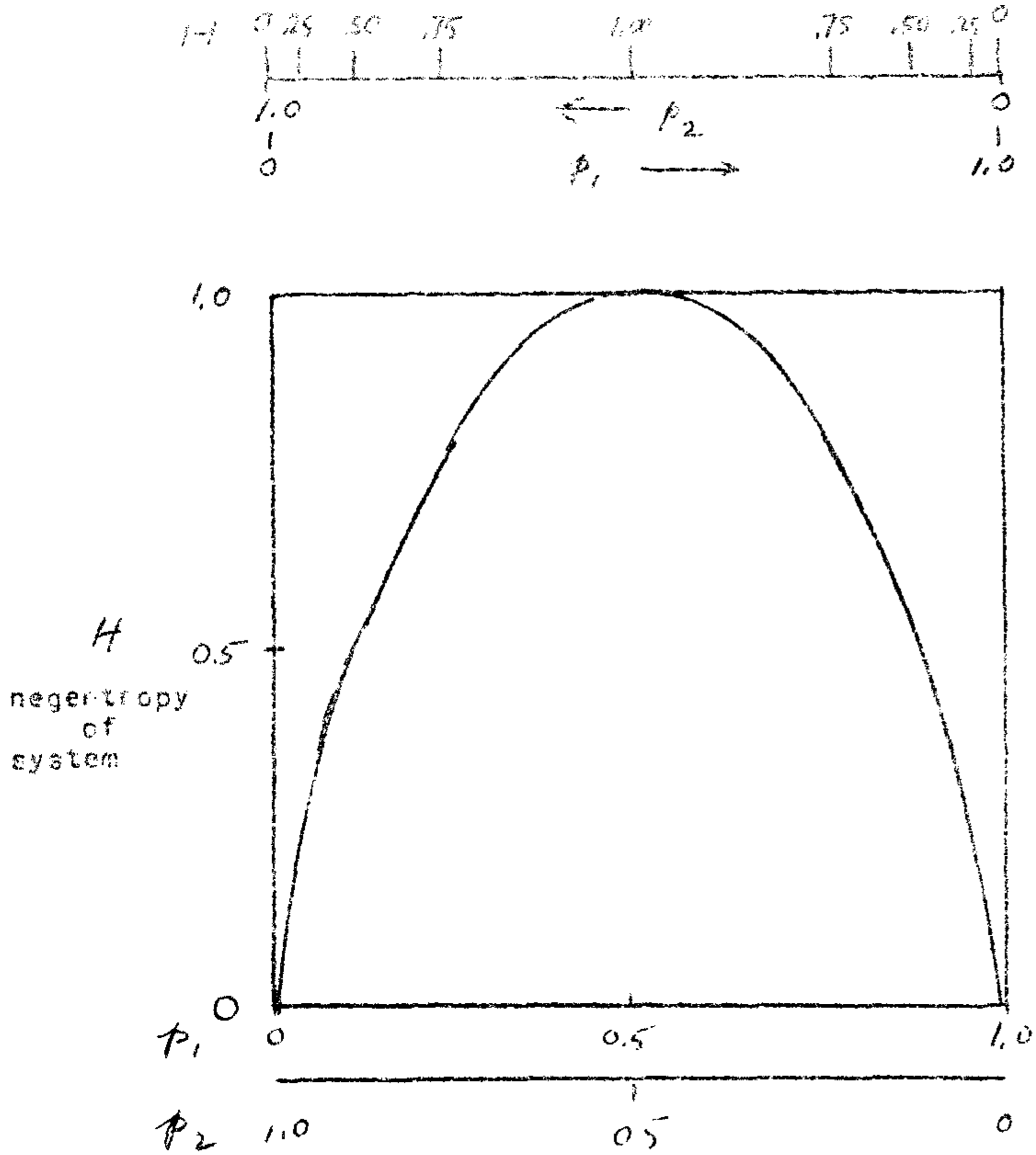
TABLE II: NEGENTROPY COMPONENTS

i	Probability p_i	Uncertainty $-U_i = \log_2 p_i$	Negentropy Component $U_i p_i$
1	0.9999930	-0.0000101	0.0000101
2	0.9999900	-0.0000144	0.0000144
3	0.9999700	-0.0000433	0.0000433
4	0.9999500	-0.0000721	0.0000721
5	0.9999300	-0.0001010	0.0001010
6	0.9999000	-0.0001443	0.0001443
7	0.9997000	-0.0004329	0.0004328
8	0.9995000	-0.0007215	0.0007212
9	0.9993000	-0.0010103	0.0010095
10	0.9990000	-0.0014434	0.0014420
11	0.9970000	-0.0043346	0.0043216
12	0.9950000	-0.0072316	0.0071954
13	0.9930000	-0.0101344	0.0100634
14	0.9900000	-0.0144996	0.0143546
15	0.9700000	-0.0439434	0.0426251
16	0.9500000	-0.0740006	0.0703006
17	0.9300000	-0.1046974	0.0973686
18	0.9000000	-0.1520032	0.1368028
19	0.7000000	-0.5145733	0.3602013
20	0.5000000	-1.0000003	0.5000001
21	0.3000000	-1.7369660	0.5210898
22	0.1000000	-3.3219289	0.3321929
23	0.0700000	-3.8365022	0.2685551
24	0.0500000	-4.3219292	0.2160965
25	0.0300000	-5.0588949	0.1517668
26	0.0100000	-6.6438578	0.0664386
27	0.0070000	-7.1584312	0.0501090
28	0.0050000	-7.6438580	0.0382193
29	0.0030000	-8.3808239	0.0251425
30	0.0010000	-9.9657867	0.0099658
31	0.0007000	-10.4803600	0.0073363
32	0.0005000	-10.9657869	0.0054829
33	0.0003000	-11.7027527	0.0035108
34	0.0001000	-13.2877157	0.0013288
35	0.0000700	-13.8022889	0.0009662
36	0.0000500	-14.2877158	0.0007144
37	0.0000300	-15.0246818	0.0004507
38	0.0000100	-16.6096444	0.0001661
39	0.0000070	-17.1242177	0.0001199

Fig 1. Probability (A), Log Probability (B) and Negentropy Component (C).

Curve (C) = Curve (A) x Curve (B)

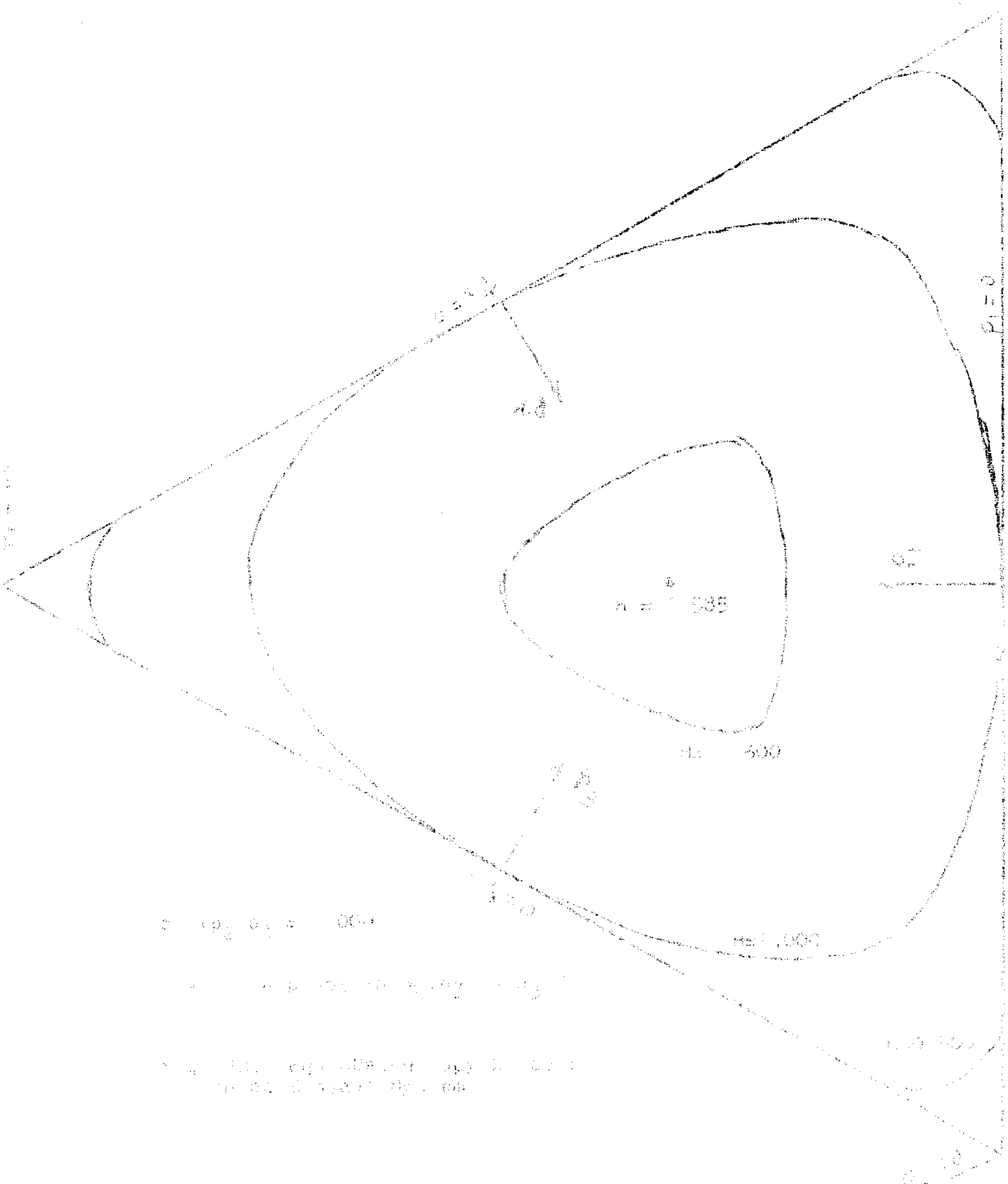




$$H = -[\, p_1 \ln p_1 + p_2 \ln p_2 \,],$$

$$p_1 + p_2 = 1.0$$

Fig. 2A. Negentropy of Two Message System.


n = 535
500
β = 0
1,000

In a similar way the triangular coordinate system for the three-message system forms the four faces of the quadrilateral cube with quadrangular coordinates. In this case equi-negentropy surfaces for H = 0, 0.5, 1.0, 1.5 & 2.0 are shown. For larger sets of messages the equi-negentropy surfaces would be in n-coordinate, n-1 space which is hard to visualize for $n > 4$. The conditions for maximum negentropy can be extended to give for the n-message case: $p_1 = p_2 = \ldots\ldots = p_n = \frac{1}{n}$ (7)

and the condition holds that: $\sum_{i=1}^{n} p_i = 1.0$ (6)

Three sample distributions corresponding to maximum negentropy are shown in Fig 3A. The cases for n=2 and n=4 cprrespond to the centers of Figs. 2A and 2C. These distributions will be used for reference when attempting to find an analogy of negentropy to use as a measure of democracy.

Another case of interest in future extensions of the concepts considered in this paper is the continous channel where there is a continous range of analog messages instead of a finite set of discrete messages. In this case eq. (4) becomes

$$H = \int p(x) \log_2 p(x)\, dx \qquad (8)$$

For an electrical signal carrying a message on a telegraph line with an average power of σ^2 and there is random noise on the line, we have a theorem from Information Theory that the negentropy is a maximum when the message distribution is gaussian, i.e.,

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \quad e^{-(x^2/2\sigma^2)} \qquad (9)$$

The equivalent condition to eq. (6) is

$$\int p(x)\, dx = 1.0 \qquad (10)$$

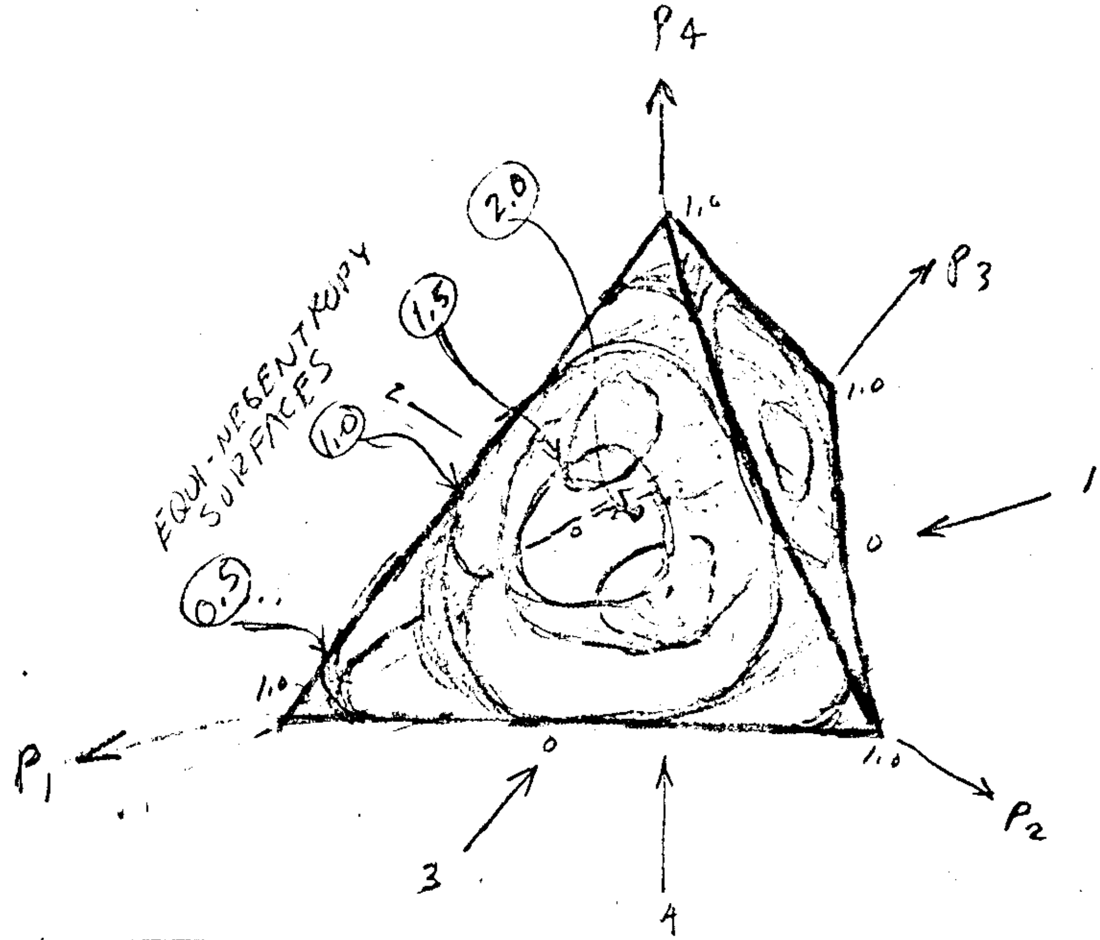


Fig. 2C. Equi-Negentropy Surfaces for Four-Message System.

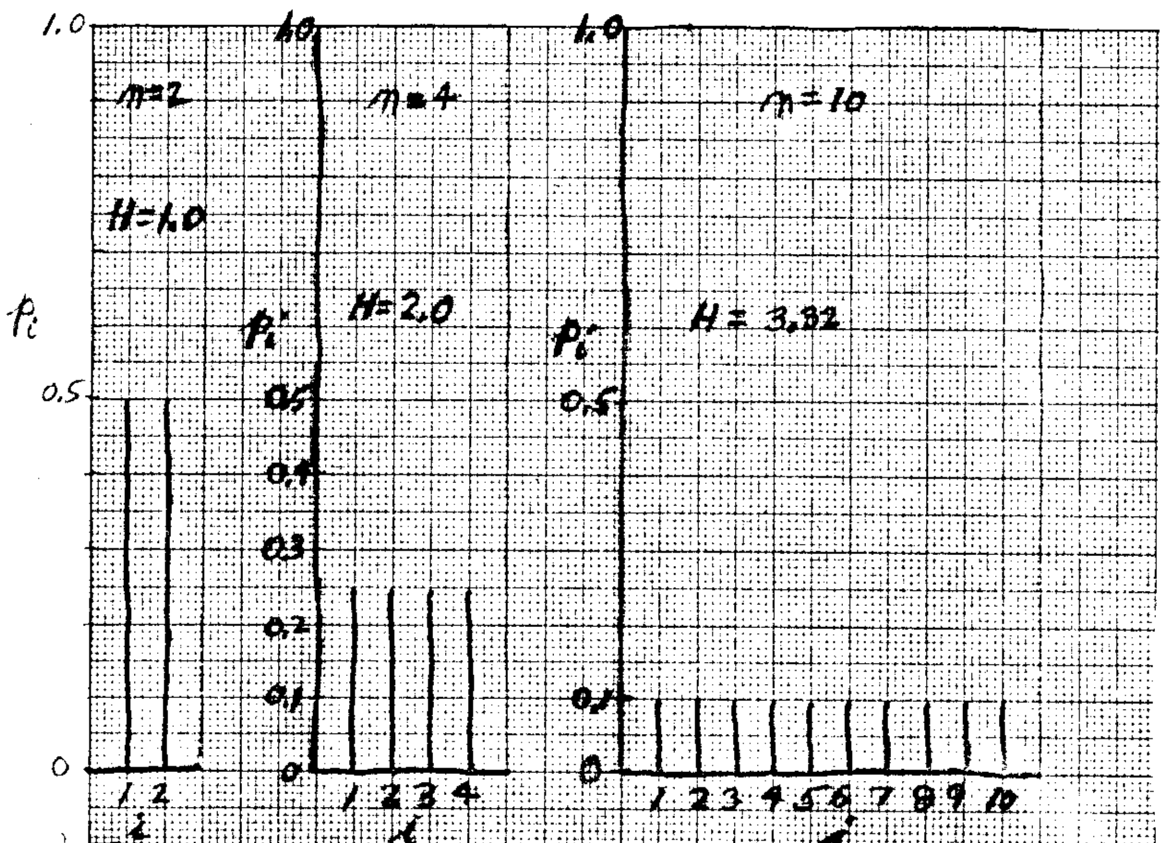


Fig. 3A Sample Distribution of Message Probabilities for n=2,4,10. (Discrete Noiseless Channel) Maximum Negentropy.

Two sets of curves are included in Fig. 3B to show sample continuous probability distributions and also power distributions, i.e., $P(x) = \sigma^2 \cdot p(x)$. The p(x) curves give the probability of messages in the range 0 to 100 occuring and satisfy eq. (10). The power distribution curves satisfy

$$\int P(x)\, dx = \sigma^2 \qquad (11).$$

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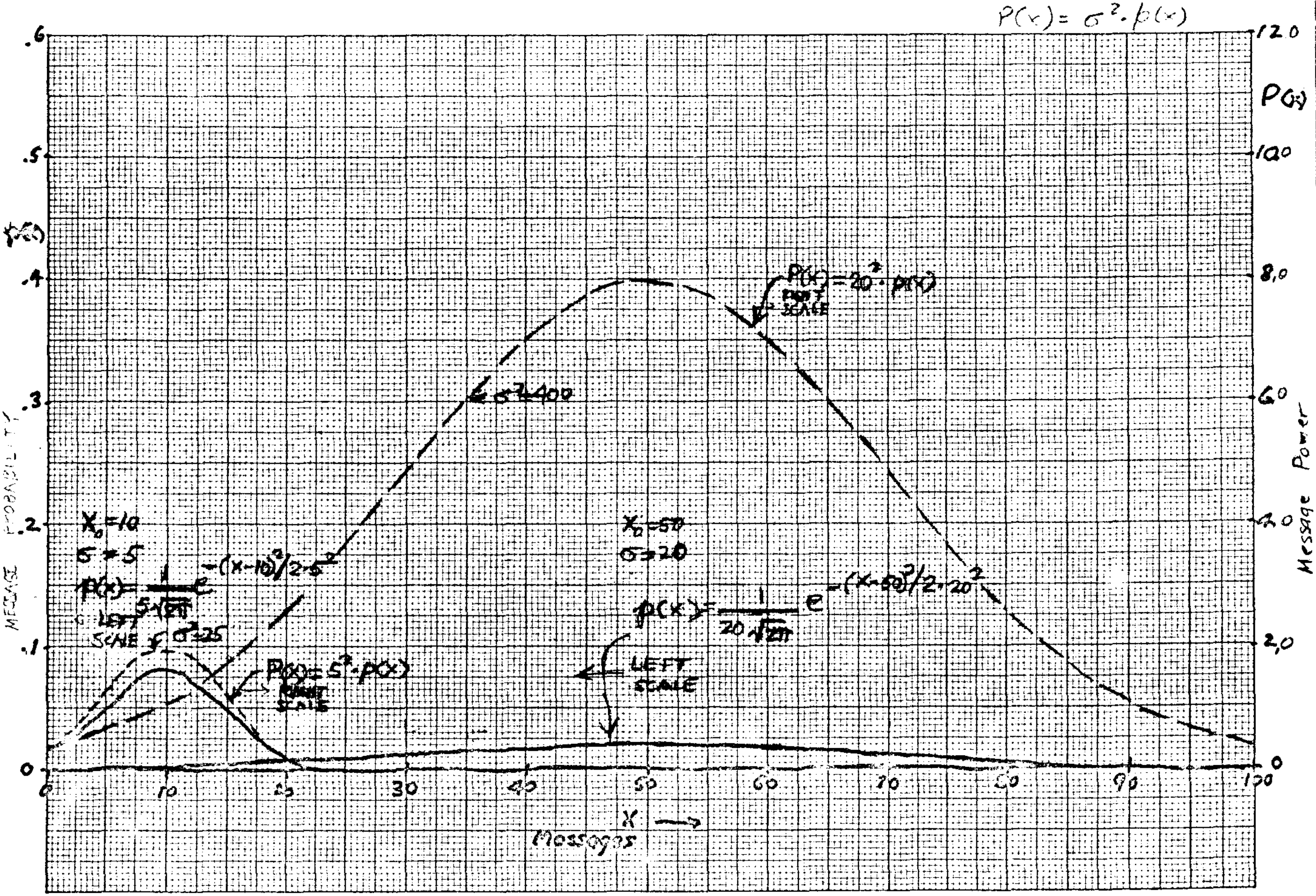

Fig. 3B Sample Distributions of $p(x)$ and $\sigma^2 p(x)$.