

## 2.0 Signal Space ( $\Sigma$ )

$$(\Sigma) = (S_1, \dots, S_i, \dots, S_m)$$

signal vector

$$S_i = [a_{i1}, a_{i2}, \dots, a_{ik}]$$

## 2.1 Bit Signal Space

Considering the signal space  $\Sigma$  for a business data transmission code such as the Transcawa code or 4-out-of-8 code (see p 2-2), leads to the subdivision of the signal space by bit and by characters. For example, the letter "A" is represented as:

$$S_A = S_A(t) = S(t) \begin{bmatrix} a_{A1} \\ a_{A2} \\ a_{A4} \\ a_{A7} \\ a_{A8} \\ a_{A9} \\ a_{A10} \\ a_{A11} \end{bmatrix} = S(t) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad (2.1)$$

↑  
bit signal  
widely

The "bit signal" may be of numerous forms:

$$S^c(t) = \cos \frac{2\pi t}{T} \quad \text{for } 0 \leq t \leq T \quad (2.2)$$

$$S^{hc}(t) = \cos \frac{\pi t}{T} \quad \text{for } 0 \leq t \leq T \quad (2.3)$$

$$S^s(t) = \sin \frac{2\pi t}{T} \quad \text{for } 0 \leq t \leq T \quad (2.4)$$

Note that the extension of Fig 1-1 to the transmission line case requires a modification of " $\otimes$ " to account for the delay characteristics.

MOLLERITH PUNCHED CARD	HOLLERITH CODE	TRANSCRV CODE	BANAC CODE	EDPM CODE	720 MAGNETIC Tape	DRUM D-PH	TAPE CODE PAR	650 LANDTYPE		
								005	702 6	703*
<b>CODE</b>										
2-0	-1	A0X0000	00X000	X00000	0X	00000	X0	0X000	91	A0X0000
2-2	-2	A0X0000	00X000	X00000	0X	00000	X0	0X000	92	00X0000
3-3	-3	00X0000	00X000	X00000	0X	00000	X0	0X000	93	00X0000
4-4	-4	XXX0000	000000	X00000	0X	00000	X0	0X000	94	00X0000
5-5	-5	00X0000	000000	X00000	0X	00000	X0	0X000	95	00X0000
6-6	-6	00X0000	000000	X00000	0X	00000	X0	0X000	96	00X0000
7-7	-7	00X0000	000000	X00000	0X	00000	X0	0X000	97	00X0000
8-8	-8	00X0000	000000	X00000	0X	00000	X0	0X000	98	00X0000
9-9	-9	00X0000	000000	X00000	0X	00000	X0	0X000	99	00X0000
10-0	-0	00X0000	000000	X00000	0X	00000	X0	0X000	00	00X0000
11-1	1-1	00X0000	000000	X00000	0X	00000	X0	0X000	01	00X0000
12-2	1-2	00X0000	000000	X00000	0X	00000	X0	0X000	02	00X0000
13-3	1-3	00X0000	000000	X00000	0X	00000	X0	0X000	03	00X0000
14-4	1-4	00X0000	000000	X00000	0X	00000	X0	0X000	04	00X0000
15-5	1-5	00X0000	000000	X00000	0X	00000	X0	0X000	05	00X0000
16-6	1-6	00X0000	000000	X00000	0X	00000	X0	0X000	06	00X0000
17-7	1-7	00X0000	000000	X00000	0X	00000	X0	0X000	07	00X0000
18-8	1-8	00X0000	000000	X00000	0X	00000	X0	0X000	08	00X0000
19-9	1-9	00X0000	000000	X00000	0X	00000	X0	0X000	09	00X0000
20-0	0-0	00X0000	000000	X00000	0X	00000	X0	0X000	00	00X0000
21-1	0-1	00X0000	000000	X00000	0X	00000	X0	0X000	01	00X0000
22-2	0-2	00X0000	000000	X00000	0X	00000	X0	0X000	02	00X0000
23-3	0-3	00X0000	000000	X00000	0X	00000	X0	0X000	03	00X0000
24-4	0-4	00X0000	000000	X00000	0X	00000	X0	0X000	04	00X0000
25-5	0-5	00X0000	000000	X00000	0X	00000	X0	0X000	05	00X0000
26-6	0-6	00X0000	000000	X00000	0X	00000	X0	0X000	06	00X0000
27-7	0-7	00X0000	000000	X00000	0X	00000	X0	0X000	07	00X0000
28-8	0-8	00X0000	000000	X00000	0X	00000	X0	0X000	08	00X0000
29-9	0-9	00X0000	000000	X00000	0X	00000	X0	0X000	09	00X0000
30-0	0-0	00X0000	000000	X00000	0X	00000	X0	0X000	00	00X0000
31-U	0-1	00X0000	000000	X00000	0X	00000	X0	0X000	01	00X0000
32-V	0-2	00X0000	000000	X00000	0X	00000	X0	0X000	02	00X0000
33-W	0-3	00X0000	000000	X00000	0X	00000	X0	0X000	03	00X0000
34-X	0-4	00X0000	000000	X00000	0X	00000	X0	0X000	04	00X0000
35-Y	0-5	00X0000	000000	X00000	0X	00000	X0	0X000	05	00X0000
36-Z	0-6	00X0000	000000	X00000	0X	00000	X0	0X000	06	00X0000
37-1	0-7	00X0000	000000	X00000	0X	00000	X0	0X000	07	00X0000
38-2	0-8	00X0000	000000	X00000	0X	00000	X0	0X000	08	00X0000
39-3	0-9	00X0000	000000	X00000	0X	00000	X0	0X000	09	00X0000
40-4	0-0	00X0000	000000	X00000	0X	00000	X0	0X000	00	00X0000
41-5	1-1	00X0000	000000	X00000	0X	00000	X0	0X000	01	00X0000
42-6	1-2	00X0000	000000	X00000	0X	00000	X0	0X000	02	00X0000
43-7	1-3	00X0000	000000	X00000	0X	00000	X0	0X000	03	00X0000
44-8	1-4	00X0000	000000	X00000	0X	00000	X0	0X000	04	00X0000
45-9	1-5	00X0000	000000	X00000	0X	00000	X0	0X000	05	00X0000
46-0	1-0	00X0000	000000	X00000	0X	00000	X0	0X000	00	00X0000
47-1	2-1	00X0000	000000	X00000	0X	00000	X0	0X000	01	00X0000
48-2	2-2	00X0000	000000	X00000	0X	00000	X0	0X000	02	00X0000
49-3	2-3	00X0000	000000	X00000	0X	00000	X0	0X000	03	00X0000
50-4	2-4	00X0000	000000	X00000	0X	00000	X0	0X000	04	00X0000
51-5	2-5	00X0000	000000	X00000	0X	00000	X0	0X000	05	00X0000
52-6	2-6	00X0000	000000	X00000	0X	00000	X0	0X000	06	00X0000
53-7	2-7	00X0000	000000	X00000	0X	00000	X0	0X000	07	00X0000
54-8	2-8	00X0000	000000	X00000	0X	00000	X0	0X000	08	00X0000
55-9	2-9	00X0000	000000	X00000	0X	00000	X0	0X000	09	00X0000
56-0	2-0	00X0000	000000	X00000	0X	00000	X0	0X000	00	00X0000
57-EOT	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
58-TEL	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
59-COL START	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
60-RESTART	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
61-PLUS ZERO	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
62-MINUS ZERO	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
63-RECORD MARK	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
64-TAPE MARK	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
65-AC & OR NR	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
66-GROUP NR	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
67-SPACE	-	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
68-12-0	12-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
69-11-0	11-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
70-10-0	10-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
71-9-0	9-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
72-8-0	8-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
73-7-0	7-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
74-6-0	6-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
75-5-0	5-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
76-4-0	4-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
77-3-0	3-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
78-2-0	2-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
79-1-0	1-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
80-0-0	0-0	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
81-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
82-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
83-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
84-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
85-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
86-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
87-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
88-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
89-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
90-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
91-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
92-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
93-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
94-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
95-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
96-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
97-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
98-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
99-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
100-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
101-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	-
102-00000	00000	00X0000	000000	X00000	0X	00000	X0	0X000	-	

The probability density function has two components: (1) bit probability and (2) character probability. The bit probabilities for the 4-out-of-8 code are:

$$\sigma(a_{ij} = 1) = 0.5$$

$$\sigma(a_{ij} = 0) = 0.5$$

## 2.2 Character Space $\Omega$

Using the 4-out-of-8 code as an example, the probability distribution functions are:

Alphabetic:  $\sigma_A'(s_i)$  where  $\sum_{i=1}^{26} \sigma_A'(s_i) = 1.0$

Determined from tables of usage of English letters in writing.

Numeric:  $\sigma_N'(s_j)$  where  $\sum_{j=0}^9 \sigma_N'(s_j) = 1.0$

Unless facts show some weighting exist, assume equal probability for each numeric.

$$\sigma_N'(s_j) = 0.1$$

Control Code:  $\sigma'_c(s_h)$  where  $\sum_{h=1}^{18} \sigma'_c(s_h) = 1.0$   
 (also special symbols)

As a first approximation the proportion of usage of special symbols + control code symbols can be estimated from present transceiver operations.

Unused (Reserve) Symbols:-

$$\sigma'_u(s_e) \text{ where } \sum_{e=1}^{16} \sigma'_u(s_e) = 1.0$$

where the probabilities of use of classes of symbols are defined as follows:

$\alpha_A$  = Probability of alphabetic symbol occurring

$\alpha_N$  = " " numeric " "

$\alpha_c$  = " " control code or special symbol occurring

$\alpha_U$  = " " unused or reserve symbol occurring

Then the character probabilities are related by:

$$\alpha_A \sum_{i=1}^{26} \sigma'_A(s_i) + \alpha_N \sum_{j=0}^9 \sigma'_N(s_j) + \alpha_c \sum_{h=1}^{18} \sigma'_c(s_h) + \alpha_U \sum_{e=1}^{16} \sigma'_u(s_e) = 1.0$$

Alternatively let  $\sigma_m = (\alpha_A \sigma'_A(s_1), \alpha_A \sigma'_A(s_2), \dots, \alpha_U \sigma'_U(s_{16}))$

then  $\sum_{m=1}^{70} \sigma_m = 1.0$

Question: Can these signal probability function be profitably put in matrix form?

Ref.: TK 3226 K73

Kazuo Konishi 2d. Memoirs of the unifying study of the basic problems in engineering science by means of geometry. v. 1.  
Tokyo (1955)

TK 3226 L36

P. Le Corbeiller Matrix Analysis of Electrical Networks Harvard (1950)

TK 3226 K8

Gabriel Kron. Tensor Analysis of Networks Wiley (1939)  
Chapt IX - Compound Tensors.

TK 2000 K75

Gabriel Kron Equivalent Circuits of Electrical Machinery Wiley (1951)

AD-63074 Bendix Aviation Corp.

Information

§68 §4.3 The Information Channel as Matrix Networks channel:

$$\vec{C}H = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$P70 \quad \vec{P} \cdot \vec{C}H = \vec{Q}$$

$$\left[ \frac{1}{2} \frac{1}{4} \frac{1}{4} \frac{1}{8} \right] \cdot \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} & 0 & 0 \end{bmatrix} = \left[ \frac{5}{32} \frac{11}{32} \frac{10}{32} \frac{6}{32} \right]$$

2-6

5-29-57

Question: Can a three dimensional  
probability matrix be developed?  
I.E. can the bit probabilities and  
character coding be introduced in the  
third dimension?

FBW